Stagnation Flow of a Jeffrey Fluid over a Shrinking Sheet

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The present paper describes the analytical solutions for the steady boundary layer flow of a Jeffrey fluid over a shrinking sheet. The governing equations of motions are reduced into a set of nonlinear ordinary differential equations by using similarity transformations. Two types of problems, namely, (1) two-dimensional stagnation flow towards a shrinking sheet and (2) axisymmetric stagnation flow towards an axisymmetric shrinking sheet, have been discussed. The series solutions of the problems are obtained by using the homotopy analysis method (HAM). The convergence of the obtained series solutions are analyzed and discussed in detail through graphs for various parameters of interest.

Key words: Stagnation Flow; Jeffrey Fluid; Shrinking Sheet; Series Solutions.

1. Introduction

The study of non-Newtonian fluids have gained considerable importance in the last years because the classical Navier-Stokes theory does not describe all the rheological properties of complex fluids such as polymer solutions, blood, paints, certain oils, greases, food mixing and chyme movement in the intestine, flow of plasma, flow of nuclear fuel slurries, flow of liquid metals and alloys, flow of mercury amalgams, and lubrication. There are two main difficulties occuring in the non-Newtonian fluid. The first difficulty is that an additional nonlinear term appears in the equations of motion rendering the problem more complex. The second difficulty is that a universal non-Newtonian constitute relation that can be used for all fluids and flows is not available. Hence, due to complexity of fluids, there are many models of non-Newtonian fluids [1 – 15]. Amongst these the Jeffrey model is a relatively simple model for which one can reasonably hope to obtain an analytical solution. Important studies to the topic include the works in [16 – 18]. Further, the flow describing the fluid motion in the neighbourhood of a stagnation line, known as stagnation point flow, has attracted many investigators during the past several decades [19 – 24].

Recently, the interest in the boundary layer flows which describe the stagnation flows and stretching (shrinking) surfaces has gained considerable importance because of their applications in industry. Such applications include rotating blades, cooling of silicon wafers, and the extrusion of polymers in a melt spinning process. The extrudate from the die is generally drawn and simultaneously stretched into a sheet which is then solidified through a quenching or gradual cooling by direct contact with water. The steady stagnation point flow towards a permeable vertical surface for both assisting and opposing flows have been discussed by Ishak et al. [25]. The steady two-dimensional mixed convection flow of an incompressible viscous fluid near an oblique stagnation point on a heated or cooled stretching vertical flat plate have been studied by Yian et al. [26]. Mahapatra and Gupta [27] have discussed the magnetohydrodynamic stagnation point flow towards a stretching sheet. The steady and unsteady boundary layer flow in the region of forward stagnation point on a stretching sheet have been examined by Nazar et al. [28].

Morerecently, Wang [29] has discussed the stagnation flow towards a shrinking sheet. In his study, he claimed that the stagnation flows are in general not aligned but previous authors considered aligned cases. He also observed that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. However, in the presence of the stagnation point the shrinking solution exists.

The aim of the present paper is to discuss the stagnation point flow of a Jeffrey fluid towards a shrinking sheet for the two-dimensional case and an axisymmetric stagnation flow towards an axisymmetric shrinking surface. To the best of authors knowledge nobody has discussed the stagnation point flow or shrinking phenomena in a Jeffrey fluid. The idea of Wang [29] has been extended for Jeffrey fluids and analytical solutions have been presented using the homotopy analysis method (HAM).

HAM is an analytical, semi-numerical technique which has been successfully applied in many fluid problems, few of them are mentioned in the References [30–44]. The solution of Wang [29] has been recovered as a special case of our problem. The convergence regions have been calculated for various physical parameters by making the h-curves. The analytical solutions have been discussed through graphs for different physical parameters appearing in the problem.

2. Formulation of the Problem

We consider the two-dimensional flow of an incompressible Jeffrey fluid near a stagnation point towards a shrinking sheet. We are choosing the cartesian coordinate system in which, u, v, w are the velocity components along x, y, and z-axis, respectively. Let the velocities $u = ax$, $w = -az$ represent the potential stagnation at infinity and $u = b(x+c)$, $w = 0$ represent the stretching (shrinking) velocities in which b is the stretching rate (shrinking if $b < 0$), $-c$ is the location of the stretching origin. The stagnation-point flows in the boundary layer over a stretching/shrinking surface are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} &= U\frac{\partial U}{\partial x} + \frac{\nu}{(1+\lambda)}\frac{\partial^2 u}{\partial z^2} + \frac{\nu\lambda_1}{(1+\lambda)} \\ &\cdot \left(u\frac{\partial^3 u}{\partial x \partial z^2} + \frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x \partial z} + w\frac{\partial^3 u}{\partial z^3} \right), \end{aligned} \tag{2}$$

where $U = ax$ is the free stream velocity, $\nu = (\mu/\rho)$ is the dynamic viscosity, ρ is the density, λ is the ratio of relaxation to retardation times, and λ_1 is the retardation time.

3. Two-Dimensional Stagnation Flow

We introduce the similarity variables and non-dimensional variables as follows [29]:

$$\begin{aligned} &\eta = \sqrt{\frac{a}{\nu}}z, \quad u = axf'(\eta) + bch(\eta), \\ &v = 0, \quad w = -\sqrt{a\nu}f(\eta). \end{aligned} \tag{3}$$

Making use of (3), the incompressibility condition is automatically satisfied and the momentum equation takes the form

$$\begin{aligned} &f''' + (1+\lambda)(ff'' - f'^2 + 1) \\ &\quad + \beta(f''^2 - ff^{\mathrm{IV}}) = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} &h'' + (1+\lambda)(fh' - f'h) \\ &\quad + ,\beta(hf''' - f'h'' + f''h' - fh''') = 0, \end{aligned} \tag{5}$$

where $\beta = a\lambda_1$.

The boundary conditions for the problem under consideration are

$$\begin{aligned} &f(0) = 0, \quad f'(0) = b/a = \alpha, \quad f'(\infty) = 1, \\ &h(0) = 1, \quad h(\infty) = 0. \end{aligned} \tag{6}$$

In the above equations the prime denotes the derivative with respect to η.

4. Axisymmetric Stagnation Flow Towards an Axisymmetric Shrinking Surface

According to Wang [29], for possible non-alignment, it is more useful to take cartesian coordinates instead of cylindrical axes. Therefore, we use the following similarity transformations for this case:

$$\begin{aligned} &\eta(x,y) = \sqrt{\frac{a}{\nu}}z, \quad u = axg'(\eta) + bcl(\eta), \\ &v = ayg'(\eta), \quad w = -2\sqrt{a\nu}g(\eta). \end{aligned} \tag{7}$$

Making use of (7), (2) takes the form

$$\begin{aligned} &g''' + (1+\lambda)(2gg'' - g'^2 + 1) \\ &\quad + \beta(g''^2 - g'g''' - 2gg^{\mathrm{IV}}) = 0, \end{aligned} \tag{8}$$

$$\begin{aligned} &l'' + (1+\lambda)(2gl' - g'l) \\ &\quad + \beta(lg''' - 2g'l'' + g''l' - 2gl''') = 0, \end{aligned} \tag{9}$$

with the corresponding boundary conditions

$$\begin{aligned} &g(0) = 0, \quad g'(0) = b/a = \alpha, \quad g'(\infty) = 1, \\ &l(0) = 1, \quad l(\infty) = 0. \end{aligned} \tag{10}$$

4.1. Solution for Two-Dimensional Stagnation Flow Towards a Shrinking Sheet by Homotopy Analysis Method

The solution of (4) and (5) has been computed by HAM. The procedure is as follows:

We select

$$f_{01}(\eta) = (1-\alpha)(\exp(-\eta) - 1) + \eta, \tag{11}$$

$$h_{01}(\eta) = \exp(-\eta) \tag{12}$$

as the initial approximations of f, h and

$$\mathcal{L}_{01}[\hat{f}(\eta;p)] = \frac{\partial^3 \hat{f}(\eta;p)}{\partial \eta^3} + \frac{\partial^2 \hat{f}(\eta;p)}{\partial \eta^2}, \quad (13)$$

$$\mathcal{L}_{02}[\hat{h}(\eta;p)] = \frac{\partial^2 \hat{h}(\eta;p)}{\partial \eta^2} + \frac{\partial \hat{h}(\eta;p)}{\partial \eta} \quad (14)$$

as auxiliary linear operators (subscript 01 means the first case and the other subscripts denote the next cases) which satisfy

$$\mathcal{L}_{01}[C_1 + C_2\eta + C_3 \mathrm{e}^{-\eta}] = 0, \quad (15)$$

$$\mathcal{L}_{02}[C_4 + C_5 \mathrm{e}^{-\eta}] = 0, \quad (16)$$

where C_i $(i = 1-5)$ are arbitrary constants. If $p \in [0,1]$ is an embedding parameter and $\hbar_i$ $(i = 1-2)$ are non-zero auxiliary parameters then the zeroth-order and mth-order deformation problems are:

Zeroth-order deformation problem

$$(1-p)\mathcal{L}_{01}[\hat{f}(\eta;p) - f_{01}(\eta)] = p\hbar_1 \mathcal{N}_1[\hat{f}(\eta;p)], \quad (17)$$

$$(1-p)\mathcal{L}_{02}[\hat{h}(\eta;p) - h_{01}(\eta)] = p\hbar_2 \mathcal{N}_2[\hat{h}(\eta;p), \hat{f}(\eta;p)], \quad (18)$$

$$\hat{f}(0;p) = 0, \quad \hat{f}'(0;p) = \alpha, \quad \hat{f}'(\infty;p) = 1, \quad (19)$$

$$\hat{h}(0;p) = 1, \quad \hat{h}(\infty;p) = 0, \quad (20)$$

where

$$\begin{aligned} \mathcal{N}_1[\hat{f}(\eta;p)] &= \frac{\partial^3 \hat{f}(\eta;p)}{\partial \eta^3} \\ &+ (1+\lambda)\left[\hat{f}(\eta;p)\frac{\partial^2 \hat{f}(\eta;p)}{\partial \eta^2} - \left(\frac{\partial \hat{f}(\eta;p)}{\partial \eta}\right)^2 + 1\right] \\ &+ \beta\left[\left(\frac{\partial^2 \hat{f}(\eta;p)}{\partial \eta^2}\right)^2 - \hat{f}(\eta;p)\frac{\partial^4 \hat{f}(\eta;p)}{\partial \eta^4}\right], \end{aligned} \quad (21)$$

$$\begin{aligned} \mathcal{N}_2[\hat{f}(\eta;p), \hat{h}(\eta;p)] &= \frac{\partial^2 \hat{h}(\eta;p)}{\partial \eta^2} \\ &+ (1+\lambda)\left[\hat{f}(\eta;p)\frac{\partial \hat{h}(\eta;p)}{\partial \eta} - \hat{h}(\eta;p)\frac{\partial \hat{f}(\eta;p)}{\partial \eta}\right] \\ &+ \beta\left[\frac{\partial \hat{h}(\eta;p)}{\partial \eta}\frac{\partial^2 \hat{f}(\eta;p)}{\partial \eta^2} + \hat{h}(\eta;p)\frac{\partial^3 \hat{f}(\eta;p)}{\partial \eta^3}\right] \\ &- \beta\left[\frac{\partial \hat{f}(\eta;p)}{\partial \eta}\frac{\partial^2 \hat{h}(\eta;p)}{\partial \eta^2} + \hat{f}(\eta;p)\frac{\partial^3 \hat{h}(\eta;p)}{\partial \eta^3}\right]. \end{aligned} \quad (22)$$

***m*th-order deformation problem**

$$\mathcal{L}_{01}[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 \mathcal{R}_{1m}(\eta), \quad (23)$$

$$\mathcal{L}_{02}[h_m(\eta) - \chi_m h_{m-1}(\eta)] = \hbar_2 \mathcal{R}_{2m}(\eta), \quad (24)$$

$$\begin{aligned} &f_m(0) = 0, \quad f'_m(0) = 0, \quad f'_m(\infty) = 0, \\ &h_m(0) = 0, \quad h_m(\infty) = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} \mathcal{R}_{1m}(\eta) &= f'''_{m-1}(\eta) + (1+\lambda)(1-\chi_m) \\ &+ (1+\lambda)\sum_{k=0}^{m-1}[f_{m-1-k}f''_k - f'_{m-1-k}f'_k] \\ &+ \beta\sum_{k=0}^{m-1}[f''_{m-1-k}f''_k - f_{m-1-k}f^{\mathrm{IV}}_k], \end{aligned} \quad (26)$$

$$\begin{aligned} \mathcal{R}_{2m}(\eta) &= h''_{m-1}(\eta) \\ &+ (1+\lambda)\sum_{k=0}^{m-1}[h'_{m-1-k}f_k - h_{m-1-k}f'_k] \\ &+ \beta\sum_{k=0}^{m-1}[h_{m-1-k}f'''_k - f'_{m-1-k}h''_k \\ &\quad + h'_{m-1-k}f''_k - f_{m-1-k}h'''_k], \end{aligned} \quad (27)$$

where

$$\chi_m = \begin{cases} 0, m \le 1, \\ 1, m > 1. \end{cases} \quad (28)$$

The symbolic software Mathematica is used to get the solutions of (23) and (24) up to the first few orders of approximations. It is found that the solution for f and h are of the following form:

$$\begin{aligned} f(\eta) &= \sum_{m=0}^{\infty} f_m(\eta) = \lim_{M\to\infty}\left[\sum_{m=0}^{M} a^0_{m,0}\right. \\ &\left. + \sum_{n=1}^{2M+1} \mathrm{e}^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=1}^{2m+1-n} a^k_{m,n}\eta^{k-1}\right)\right], \end{aligned} \quad (29)$$

$$\begin{aligned} &h(\eta) = \sum_{m=0}^{\infty} h_m(\eta) = \\ &\lim_{M\to\infty}\left[\sum_{n=1}^{2M+1} \mathrm{e}^{-(n+1)\eta}\left(\sum_{m=n-1}^{2M}\sum_{k=0}^{2m+1-n} c^k_{m,n}\eta^{k-1}\right)\right], \end{aligned} \quad (30)$$

in which the coefficients $a^q_{m,n}$ and $c^q_{m,n}$ of $f_m(\eta)$ and $h_m(\eta)$ can be found by using the given boundary conditions and by initial guess approximations in (11) and (12). The numerical data of the above mentioned solutions have been presented through graphs. The h-curves see in Figures 1 and 2.

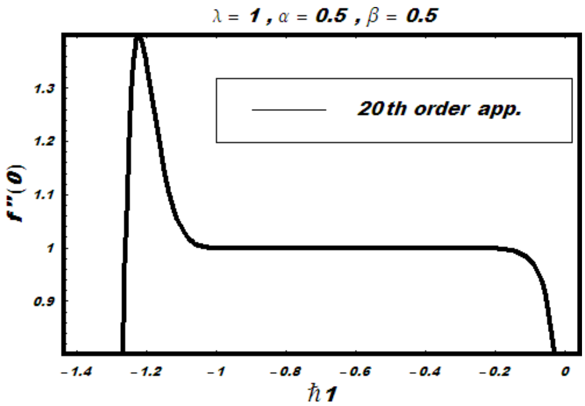

Fig. 1. h-curve of f for two-dimensional stagnation flow.

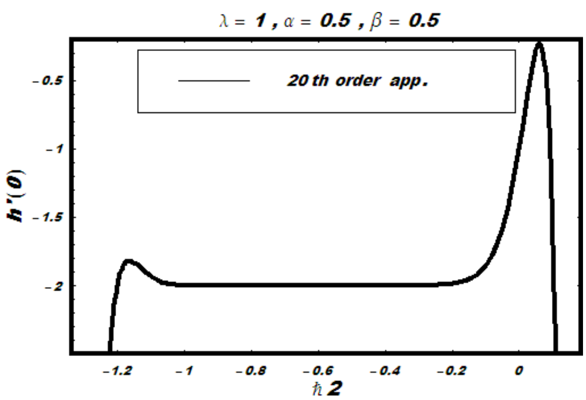

Fig. 2. h-curve of h for two-dimensional stagnation flow.

4.2. Axisymmetric Stagnation Flow Towards an Axisymmetric Shrinking Surface

In order to find the HAM solution of (8) and (9), we select

$$g_{02}(\eta) = (1-\alpha)(\exp(-\eta)-1)+\eta, \tag{31}$$

$$l_{02}(\eta) = \exp(-\eta) \tag{32}$$

as the initial approximations of g, l and

$$\mathcal{L}_{04}[\hat{g}(\eta;p)] = \frac{\partial^3 \hat{g}(\eta;p)}{\partial\eta^3} + \frac{\partial^2 \hat{g}(\eta;p)}{\partial\eta^2}, \tag{33}$$

$$\mathcal{L}_{05}[\hat{l}(\eta;p)] = \frac{\partial^2 \hat{l}(\eta;p)}{\partial\eta^2} + \frac{\partial \hat{l}(\eta;p)}{\partial\eta} \tag{34}$$

as auxiliary linear operators (subscript 01 means the first case and the other subscripts denote the next cases), which satisfy

$$\mathcal{L}_{04}[C_6 + C_7\eta + C_8 \mathrm{e}^{-\eta}] = 0, \tag{35}$$

$$\mathcal{L}_{05}[C_9 + C_{10}\mathrm{e}^{-\eta}] = 0, \tag{36}$$

where C_i $(i = 6-10)$ are arbitrary constants. If $p \in [0,1]$ is an embedding parameter and $\hbar_i$ $(i = 3-4)$ are non-zero auxiliary parameters then the zeroth-order and mth-order deformation problems are:

Zeroth-order deformation problem

$$\begin{aligned}&(1-p)\mathcal{L}_{04}[\hat{g}(\eta;p) - g_{02}(\eta)]\\ &= p\hbar_4 \mathcal{N}_4[\hat{g}(\eta;p)],\end{aligned} \tag{37}$$

$$\begin{aligned}&(1-p)\mathcal{L}_{05}[\hat{l}(\eta;p) - l_{02}(\eta)]\\ &= p\hbar_5 \mathcal{N}_5[\hat{l}(\eta;p),\hat{g}(\eta;p)],\end{aligned} \tag{38}$$

$$\hat{g}(0;p) = 0, \quad \hat{g}'(0;p) = \alpha, \quad \hat{g}'(\infty;p) = 1, \tag{39}$$

$$\hat{l}(0;p) = 1, \quad \hat{l}(\infty;p) = 0, \tag{40}$$

where

$$\begin{aligned}\mathcal{N}_3[\hat{g}(\eta;p)] &= \frac{\partial^3 \hat{g}(\eta;p)}{\partial\eta^3} + (1+\lambda)\Bigg[2\hat{g}(\eta;p)\frac{\partial^2 \hat{g}(\eta;p)}{\partial\eta^2}\\ &- \left(\frac{\partial \hat{g}(\eta;p)}{\partial\eta}\right)^2 + 1\Bigg] + \beta\Bigg[\left(\frac{\partial^2 \hat{g}(\eta;p)}{\partial\eta^2}\right)^2\\ &- \frac{\partial \hat{g}(\eta;p)}{\partial\eta}\frac{\partial^3 \hat{g}(\eta;p)}{\partial\eta^3} - 2\hat{g}(\eta;p)\frac{\partial^4 \hat{g}(\eta;p)}{\partial\eta^4}\Bigg],\end{aligned} \tag{41}$$

$$\begin{aligned}\mathcal{N}_4[\hat{g}(\eta;p),\hat{l}(\eta;p)] &= \frac{\partial^2 \hat{l}(\eta;p)}{\partial\eta^2}\\ &+ (1+\lambda)\left[2\hat{g}(\eta;p)\frac{\partial \hat{l}(\eta;p)}{\partial\eta} - \hat{l}(\eta;p)\frac{\partial \hat{g}(\eta;p)}{\partial\eta}\right]\\ &+ \beta\left[\frac{\partial \hat{l}(\eta;p)}{\partial\eta}\frac{\partial^2 \hat{g}(\eta;p)}{\partial\eta^2} + \hat{l}(\eta;p)\frac{\partial^3 \hat{g}(\eta;p)}{\partial\eta^3}\right]\\ &- 2\beta\left[\frac{\partial \hat{g}(\eta;p)}{\partial\eta}\frac{\partial^2 \hat{l}(\eta;p)}{\partial\eta^2} + \hat{g}(\eta;p)\frac{\partial^3 \hat{l}(\eta;p)}{\partial\eta^3}\right].\end{aligned} \tag{42}$$

***m*th-order deformation problem**

$$\mathcal{L}_{04}[g_m(\eta) - \chi_m g_{m-1}(\eta)] = \hbar_4 \mathcal{R}_{4m}(\eta), \tag{43}$$

$$\mathcal{L}_{05}[l_m(\eta) - \chi_m l_{m-1}(\eta)] = \hbar_5 \mathcal{R}_{5m}(\eta), \tag{44}$$

$$\begin{aligned}&g_m(0) = 0, \quad g'_m(0) = 0, \quad g'_m(\infty) = 0,\\ &l_m(0) = 0, \quad l_m(\infty) = 0,\end{aligned} \tag{45}$$

$$\begin{aligned}\mathcal{R}_{3m}(\eta) &= g'''_{m-1}(\eta) + (1+\lambda)(1-\chi_m)\\ &+ (1+\lambda)\sum_{k=0}^{m-1}[2g_{m-1-k}g''_k - g'_{m-1-k}g'_k]\\ &+ \beta\sum_{k=0}^{m-1}[g''_{m-1-k}g''_k - g'_{m-1-k}g'''_k - 2g_{m-1-k}g^{\mathrm{IV}}_k]\end{aligned} \tag{46}$$

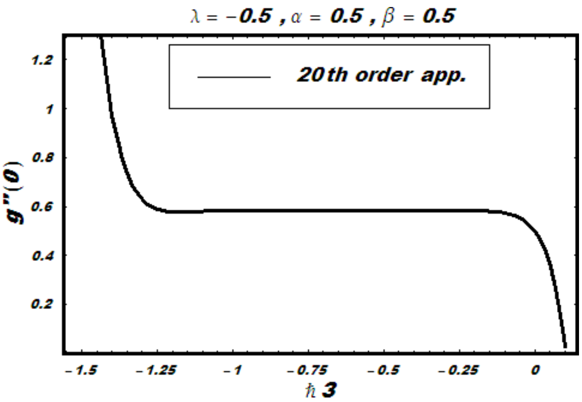

Fig. 3. h-curve of g for axisymmetric stagnation flow towards an axisymmetric shrinking surface.

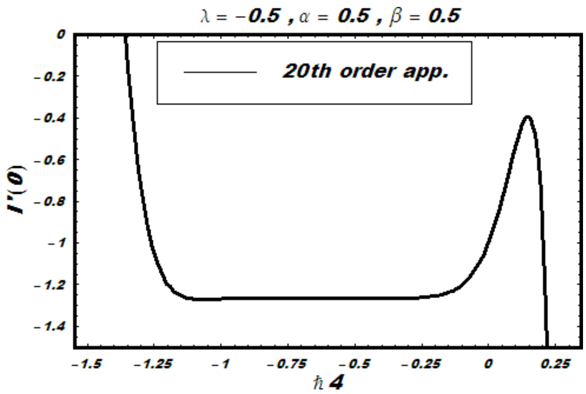

Fig. 4. h-curve of l for axisymmetric stagnation flow towards an axisymmetric shrinking surface.

$$\begin{aligned}\mathcal{R}_{4m}(\eta) &= l''_{m-1}(\eta)\\ &+(1+\lambda)\sum_{k=0}^{m-1}[2l'_{m-1-k}g_k - l_{m-1-k}g'_k]\\ &+\beta\sum_{k=0}^{m-1}[l_{m-1-k}g'''_k - 2g'_{m-1-k}l''_k + l'_{m-1-k}g''_k\\ &\qquad -2g_{m-1-k}l'''_k].\end{aligned} \tag{47}$$

The solutions of (43) and (44) up to the first few orders of approximations are defined as

$$\begin{aligned}g(\eta) &= \sum_{m=0}^{\infty} g_m(\eta) = \lim_{M\to\infty}\Bigg[\sum_{m=0}^{M} a^0_{m,0}\\ &+\sum_{n=1}^{2M+1} \mathrm{e}^{-(n+2)\eta}\Bigg(\sum_{m=n-1}^{2M}\ \sum_{k=0}^{2m+1-n} a^k_{m,n}\eta^{k-1}\Bigg)\Bigg],\end{aligned} \tag{48}$$

$$\begin{aligned}l(\eta) &= \sum_{m=0}^{\infty} l_m(\eta) =\\ &\lim_{M\to\infty}\Bigg[\sum_{n=1}^{2M+1} \mathrm{e}^{-(n+2)\eta}\Bigg(\sum_{m=n-1}^{2M}\ \sum_{k=0}^{2m+1-n} c^k_{m,n}\eta^{k-1}\Bigg)\Bigg]\end{aligned} \tag{49}$$

in which the coefficients $a^q_{m,n}$, $c^q_{m,n}$ of $g_m\Big(\eta\Big)$ and $l_m(\eta)$ are constants. The h-curves see in Figures 3 and 4.

5. Results and Discussion

The velocities f' and h for the two-dimensional stagnation flow are presented in Figures 5 – 12. The non-dimensional velocity f' and h against η for different values of stretching $\alpha > 0$ (shrinking $\alpha < 0$) are plotted in Figures 5 – 8. It is observed that f' increases with the increase in the stretching parameter

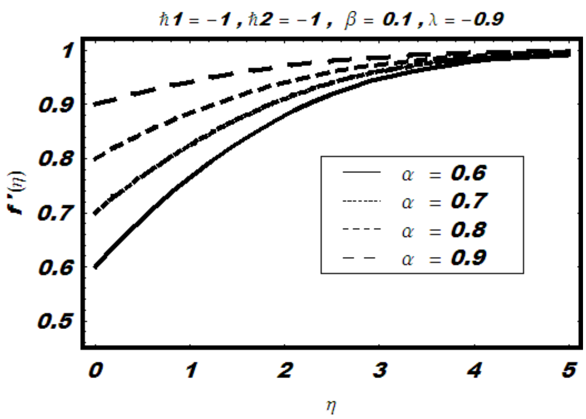

Fig. 5. Influence of stretching parameter α on f for two-dimensional stagnation flow.

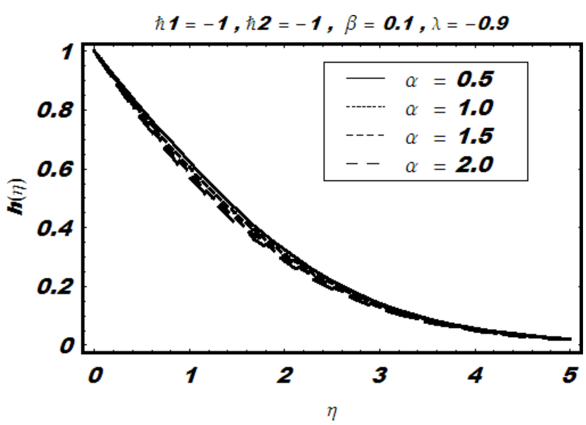

Fig. 6. Influence of stretching parameter α on h for two-dimensional stagnation flow.

($\alpha > 0$) whereas h decreases with the increase in α (see Figs. 5 and 6). The results are quite opposite for the case of shrinking ($\alpha < 0$) which is shown in Fig-

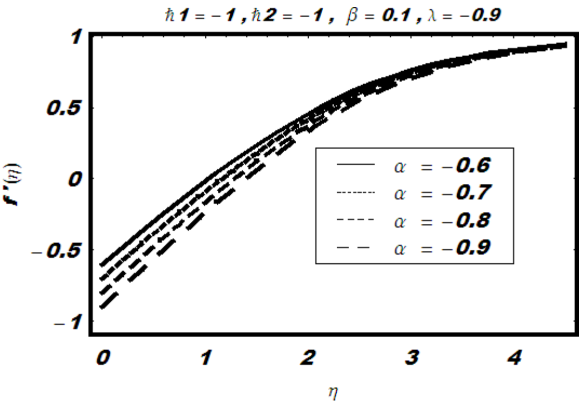

Fig. 7. Influence of shrinking parameter α on f for two-dimensional stagnation flow.

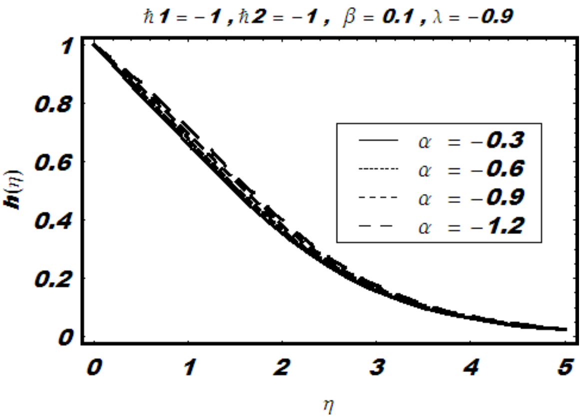

Fig. 8. Influence of shrinking parameter α on h for two-dimensional stagnation flow.

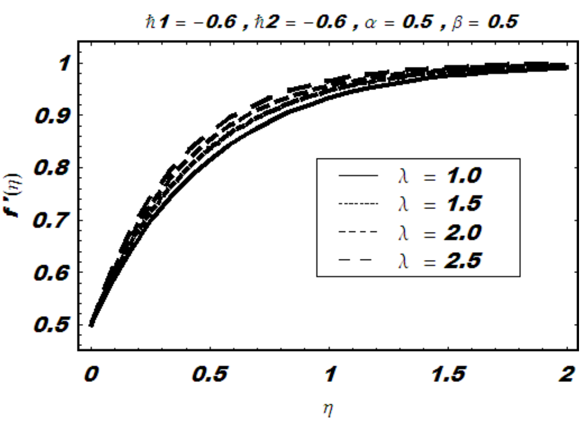

Fig. 9. Influence of λ on f for two-dimensional stagnation flow.

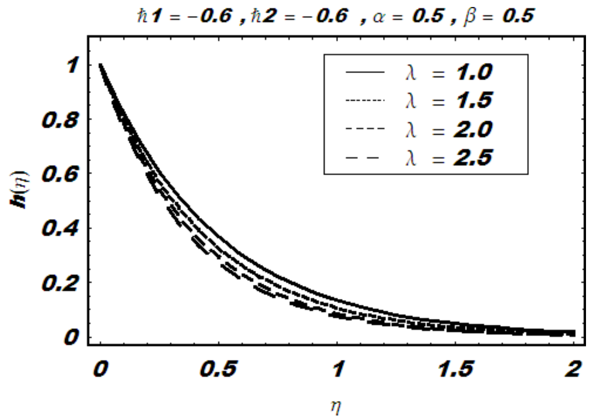

Fig. 10. Influence of λ on h for two-dimensional stagnation flow.

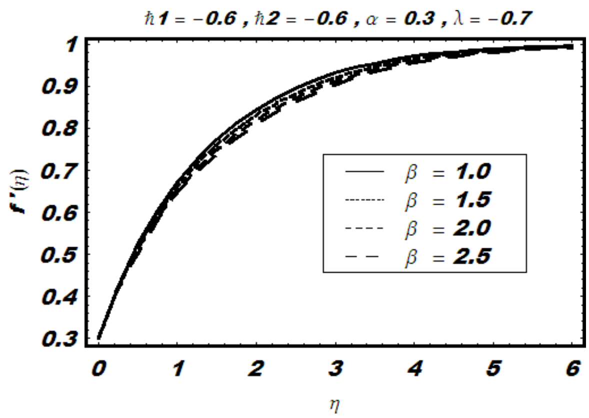

Fig. 11. Influence of β on f for two-dimensional stagnation flow.

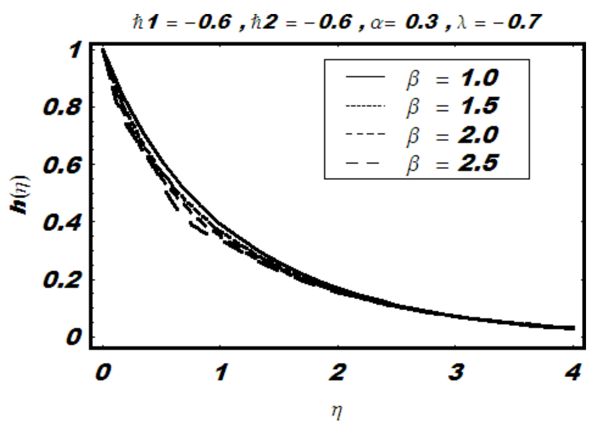

Fig. 12. Influence of β on h for two-dimensional stagnation flow.

ures 7 and 8. The effect of λ on f' and h is shown in Figures 9 and 10. It is depicted that with the increase in λ, f' increases while h decreases. The behaviour of β on f' and h is shown in Figures 11 and 12

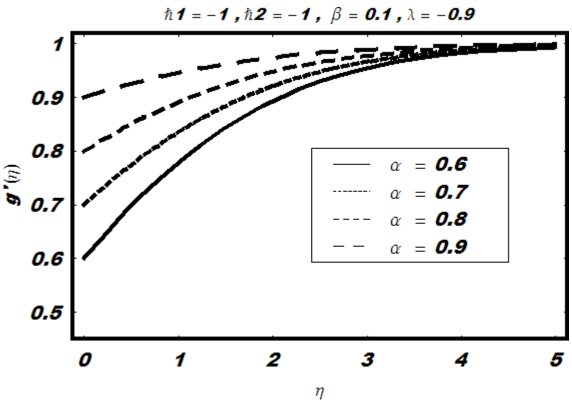

Fig. 13. Influence of stretching parameter α on g for axisymmetric stagnation flow towards axisymmetric shrinking surface.

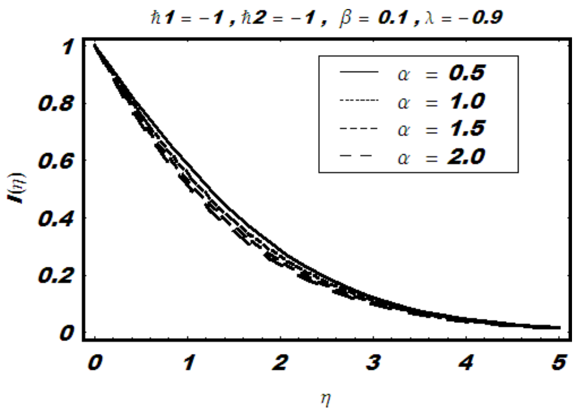

Fig. 14. Influence of stretching parameter α on h for axisymmetric stagnation flow towards axisymmetric shrinking surface.

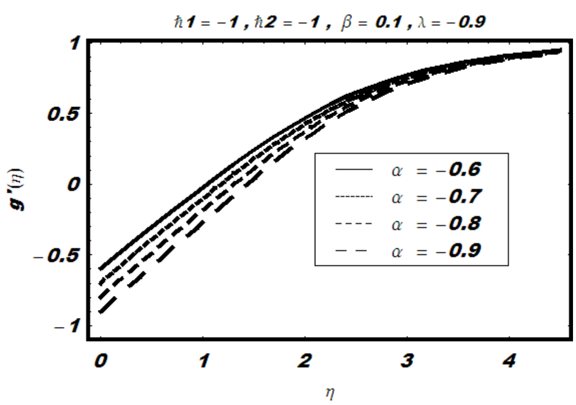

Fig. 15. Influence of shrinking parameter α on g for axisymmetric stagnation flow towards axisymmetric shrinking surface.

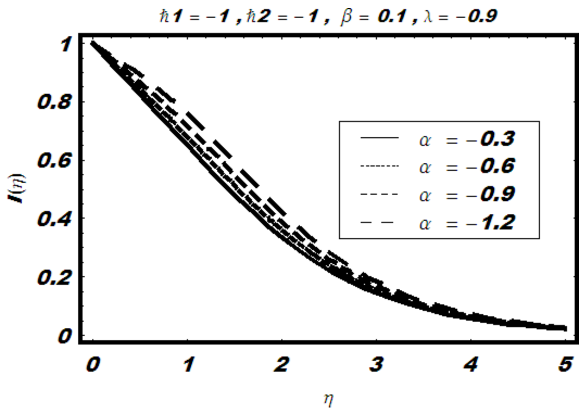

Fig. 16. Influence of shrinking parameter α on l for axisymmetric stagnation flow towards axisymmetric shrinking surface.

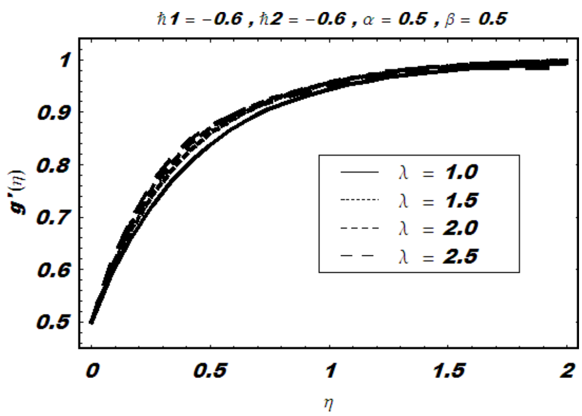

Fig. 17. Influence of λ on g for axisymmetric stagnation flow towards axisymmetric shrinking surface.

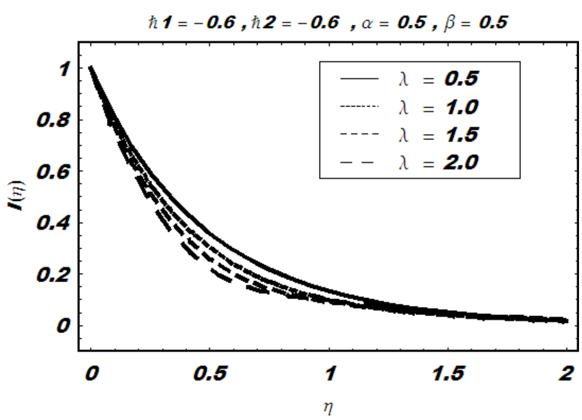

Fig. 18. Influence of λ on l for axisymmetric stagnation flow towards axisymmetric shrinking surface.

which gives decrease in f' and h with the increase in β. Figures 13 – 20 are prepared for g' and l against η for various values of α, λ, and β for axisymmetric stagnation flow on an axisymmetric shrinking surface. It is

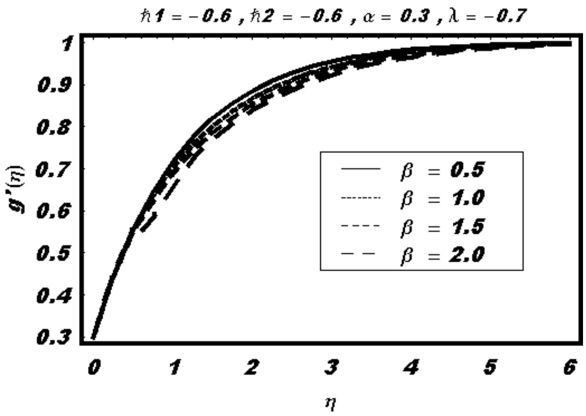

Fig. 19. Influence of β on g for axisymmetric stagnation flow towards axisymmetric shrinking surface.

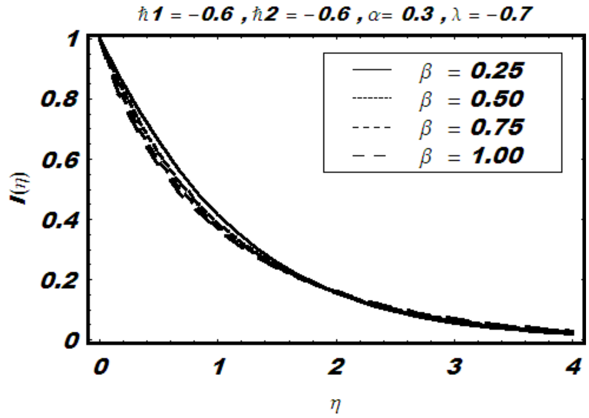

Fig. 20. Influence of β on l for axisymmetric stagnation flow towards axisymmetric shrinking surface.

shown that g' increases with the increase in the stretching parameter α, whereas l decreases with the increase in α (see Figs. 13 and 14). The behaviour of g' and l for the shrinking case is opposite to stretching case (see Figs. 15 and 16). The effect of λ on g' and l is shown in Figures 17 and 18. It is depicted that with the increase in λ, g' increases but l decreases. The behaviour of β on g' and l is shown in Figures 19 and 20 which gives decrease in g' and l with the increase in β.

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